Formation of a series of high-intensity light bullets in a femtosecond laser pulse

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Abstract: The influence of the group velocity dispersion in transparent dielectrics and the power of laser pulses on regularities in formation of a series of light bullets with high spatial localization of the optical field in a femtosecond filament has been studied numerically. It has been found that the number of light bullets in a wave packet depends on the ratio of similarity parameters, which determine self-focusing and self-compression of the laser pulse under conditions of anomalous group velocity dispersion. It has been shown that the peak intensity in a light bullet (exceeding 100 TW/cm²) is independent of the initial intensity in the pulse and increases with an increase of the number of photons involved in the process of photoionization. The conclusions obtained are generalized to the problem of transportation of high-density energy in the atmosphere by mid-IR pulses.

Keywords: filamentation, light bullet, femtosecond pulse, anomalous group velocity dispersion.

1. Introduction

The light bullet (LB) is a self-forming robust formation, whose parameters are determined by nonlinear and dispersion properties of a medium and independent of the parameters of an acting pulse. According to [1], the light bullet is formed as a result of the joint and coordinated compression of laser pulse both in space and in time. A necessary condition for the LB formation is the anomalous group velocity dispersion (AGVD). For this to take place, the pulse compression in space and in time should occur simultaneously in the process of pulse propagation in a nonlinear dispersive medium. The formation of light bullets and their sequence in process of filamentation of a femtosecond laser pulse in volume of transparent medium at a wavelength falling within the AGVD range has the same scenario in both the condensed medium and gases.

The high spatiotemporal localization of optical radiation in LB opens up new possibilities in development of methods for time-resolved diagnostics and systems for transfer of high-density laser energy. The studies of formation of light bullets in the air are oriented at the problems of environmental monitoring and sensing.

In this paper, we study numerically the formation of LB series at filamentation of femtosecond pulses in condensed media. The influence of AGVD on formation of the following LBs at filamentation of microjoule mid-IR pulses at different wavelengths in fused silica is considered. To find a waveguide mode of propagation with a long-lived LB, the filamentation of pulses at a wavelength of 3 µm with energy of several microjoules in an isotropic CaF₂ crystal is considered.

2. Model of filamentation

In the numerical simulation, we use the approximation of slowly varying wave [2], according to which the equation in the moving coordinate system for the envelope of wave packet A(r,t,z) at the carrier frequency ω₀ under conditions of axial symmetry has the form [3, 4]:

\[
2ik_0 \frac{\partial A}{\partial z} = i\hat{T}^{-1} \Delta_\perp A + \int_{-\infty}^{\infty} \frac{1}{1 + \Omega/\omega_0} \left( k^2 (\omega_0 + \Omega) - (k_0^2 + k_1^2) \right) \tilde{A}(r,\Omega,z)e^{i\Omega} d\Omega +
\]
Here, $\hat{A}(r,\Omega,z)$ is the Fourier transform of the envelope; $\Omega = \omega - \omega_0$ is the frequency shift of supercontinuum harmonics at the frequency $\omega$ from the carrier frequency $\omega_0$; $k(\omega) = \omega n(\omega)/c_0$, where $n(\omega)$ is the dispersion dependence of fused silica or CaF$_2$; $c_0$ is the speed of light in vacuum, $\Delta n_k$ is the Kerr increment of the refractive index, $k_0 = k(\omega_0)$, $k_1 = \partial k/\partial \omega|_{\omega_0}$ . The operator $\hat{T} = 1 - i\frac{\partial}{\omega_0 \partial t}$ being a result of consideration of terms having the following order of smallness in the serial expansion of the wave operator $(\partial/\partial z - ik)^2$, allows the wave packets, whose duration shortens down to one optical oscillation [2], to be considered within the framework of model (1).

The increment of the refractive index caused by generation of laser plasma is determined by the equation $\Delta n_z(r,t) = -4\pi c_0^2 N_e(r,t)/2n_0 c^2 \omega_0 m_e$, where $m_e$ and $e$ are the electron mass and charge, respectively.

The concentration of free electrons $N_e$ obeys the kinetic equation

$$\frac{\partial N_e}{\partial t} = W(|A^2|)(N_0 - N_e) + \nu_i N_e,$$

where $N_0$ is the concentration of neutral atoms; $\nu_i = (1/U_i)\left((e^2 |A|^2)/2m_e(\omega_0^2 + \nu_i^2)\right)$ is the frequency of avalanche ionization, $U_i$ is the width of forbidden zone; $\nu_i$ is the frequency of electron collisions with neutrals. The rate of field ionization $W(|A|)$ is determined by the Keldysh equation [5]. The cross section of bremsstrahlung absorption is calculated as $\sigma = (k_0 4\pi e^2/n_0^2 \omega_0^4 m_e)(\nu_i/\omega_0)$. The coefficient of attenuation caused by the field ionization of the medium has the form $\alpha(r,\tau) = (K\lambda_0 L_0^2)|A|^2(N_0 - N_e(r,\tau))$.

We consider a collimated beam of the spectrally limited pulsed radiation of the Gaussian form

$$A(r,\tau,\zeta = 0) = A_0 \exp\left[\frac{-r^2}{2a_0^2} - \frac{\tau^2}{2\tau_0^2}\right],$$

where $a_0$ and $\tau_0$ are the beam radius and the pulse half-duration at the $e^{-1}$ intensity level; $A_0$ is the peak amplitude of the optical field. The parameter $\tau_0$ is connected with the pulse duration at half-maximum $\tau_{1/2}$, which is usually measured experimentally, simply as $\tau_{1/2} = 2\ln 2 \tau_0$. The similarity parameters of the problem under consideration are the dispersion length $L_{dis} = \tau_0^2/k_2$, the diffraction length $L_{dif} = k a_0^2$, and the self-focusing length $L_{sf}$.

For the numerical solution of the system of equations (1)–(3) with the initial condition (4), we used the splitting method, in which the equations describing dispersion and diffraction of a pulse at every step along the evolution coordinate $z$ were considered with respect to the frequency $\hat{A}(r,\Omega,z)$ and spatial $A(\eta,\tau,z)$ spectra of the envelope, respectively. This has allowed us to use the dispersion relation $n(\omega)$ directly in the Sellmeier form.

Based on the solution of problem (1)–(4), we have studied the transformation of the spatiotemporal distribution of the intensity $I(r,\tau,z)$ in the process of LB formation at filamentation of femtosecond pulses of different wavelength in fused silica and the radiation at a wavelength of 3 µm and the variable power in CaF$_2$.
3. Influence of dispersion on formation of a series of light bullets

To study the role of group velocity dispersion in formation of an LB series, we have considered the filamentation of femtosecond pulses at three wavelengths 1.4, 1.8, and 2.2 μm, which fall within the AGVD range, in fused silica. To separate the influence of just the group velocity dispersion (GVD) on light bullets, the pulse half-duration \( \tau_0 \), the beam radius \( a_0 \), and the nonlinearity parameter \( R = \frac{P_{\text{peak}}}{P_{\text{cr}}} \), where \( P_{\text{peak}} = \pi a_0^2 I_0 \) is the peak power (\( I_0 \) is the initial intensity), \( P_{\text{cr}} = 3.77 \pi a_0^2 (2k_0^2 n_2) \) is the critical power for self-focusing, were taken identical for all the wavelengths. As an example, Fig. 1 shows the results of calculation for a wavelength of 1.8 μm.

![Figure 1. Pulse at a wavelength of 1.8 μm in fused silica. Spatiotemporal intensity distributions in the log scale \( \lg(I(r,t)/I_0) \) and the intensity profile at the axis \( I(r = 0, t) \) at the distance: \( Z_{\text{bul,1}}^{(1,8)} = 1.94 \) nm – the first light bullet is formed (a); \( Z = 1.97 \) cm – the bullet breaks down (b); \( Z = 2.01 \) cm – the second bullet arises (c); \( Z_{\text{bul,2}}^{(1,8)} = 2.13 \) cm – the second bullet is formed (d).

It follows from the analysis of the results obtained for pulses at wavelengths of 1.4, 1.8, and 2.2 μm that in the case of weak GVD \( (L_{\text{dis}} >> L_{\text{sf}}) \) the pulse compression at the phase self-modulation appears to be insufficient for formation of the following light bullets in the series. In the case of strong GVD \( (L_{\text{dis}} << L_{\text{sf}}) \), the decrease of the peak intensity of the pulse due to dispersion spreading before the filament formation prevents the formation of the following light bullets. As the wavelength increases, the LB diameter increases, whereas the duration shortens. The intensity clamping in LB is about \( I_{\text{peak}} = 40 \) TW/cm\(^2\) and weakly depends on wavelength, since the generation of laser plasma limiting the intensity growth has the threshold character at the number of involved photons \( K = 11–16 \).

4. Influence of pulse power on formation of light bullets in CaF\(_2\)

The influence of the peak power on LB formation was analyzed for the radiation at a wavelength of 3 μm for the duration parameter \( \tau_0 = 30 \) fs and the beam radius \( a_0 = 115 \) μm. The pulses with energy of 15.9, 10.6, 7.9, and 5.3 μJ were studied. For these pulses, the nonlinearity parameter was \( R = 3.0, 2.0, 1.5, \) and \( 1.0 \), and the self-focusing length was \( L_{\text{sf}} = 1.64, 2.65, 4.2 \) cm, and \( \infty \), respectively.

For the pulse with the nonlinearity parameter \( R = 3 \), at which the initial intensity is \( I_0 = 360 \) GW/cm\(^2\), the spatiotemporal intensity distributions \( \lg(I(r,t)/I_0) \) and the axial intensity profiles \( I(r = 0, t) \) are shown in Fig. 2 for some characteristic distances \( Z \). The first LB is formed at the distance \( Z_{\text{bul,1}}^{(3)} = 1.34 \) cm (Fig. 2, a), and at the distance \( Z_{\text{bul,2}}^{(3)} = 1.41 \) cm the second LB arises, whose plasma channel defocuses the first light bullet (Fig. 2, b). After these two LBs, existing together at a range about 2 mm long (Fig. 2, c), the peak intensity decreases more than ten times (Fig. 2, d). The third LB is formed at the distance \( Z_{\text{bul,3}}^{(3)} = 1.84 \) cm, the fourth one is formed at the distance \( Z_{\text{bul,4}}^{(3)} = 2.25 \) cm after manifold decrease of the intensity in the filament (Figs. 2, f–g), and the fifth one is formed at the distance \( Z_{\text{bul,5}}^{(3)} = 3.07 \) cm (Fig. 2, h). The values of the peak intensity \( I_{\text{peak}} \) in the
formed LBs are close and approximately equal to 120 TW/cm², the LB diameter is about 10 µm, and the duration is about 6 fs.

![Figure 2. Pulse at a wavelength of 3 µm in CaF₂.](image)

Thus, a series of short-lived LBs was formed in the filament at a length of about 1.7 cm.

**Summary**

The formation of light bullets at the femtosecond pulse filamentation under the AGVD conditions is a multifactor process and is determined by the fundamental characteristics of the pulse and the medium. The analysis of the results obtained allows us to draw some conclusions:

1. Depending on the relation between the similarity parameters, AGVD can disturb the formation of an LB series.
2. The peak intensity in LB is independent of the initial intensity in the pulse and determined by the number of photons $K$ involved in the process of ionization at the generation of laser plasma.
3. The range of LB existence is about 1 mm, which corresponds to the life time of several picoseconds.

The revealed general regularities in formation of light bullets at the pulse filamentation under conditions of anomalous group velocity dispersion allow the LB characteristics to be studied based on the common approach both in condensed media and in gases.

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5. **References**