# A Measure of Flow Vorticity with Twisted Beams of Light

**Aniceto Belmonte[1], Carmelo Rosales-Guzmán[2], and Juan P. Torres[1, 2]**
*(1) Technical University of Catalonia, BarcelonaTech, Dept. of Signal Theory and Communications*
*Campus Nord UPC, 08034 Barcelona, Spain*
*(2) ICFO–Institut de Ciencies Fotoniques*
*Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
belmonte@tsc.upc.edu

**Abstract:** The measurement of vorticity, a parameter providing local measurements of rotation at every point in a flow, would greatly assist research fields as diverse as biology microfluidics, complex motions in the oceanic and atmospheric boundary layers, and wake turbulence on fluid aerodynamics. However, the precise measurement of flow vorticity is difficult. Here, we devise an experiment in which the local vorticity of a flow can be estimated by probing the fluid with Laguerre-Gauss beams, optical beams that show an azimuthal phase variation that is the origin of its characteristic non-zero orbital angular momentum. The key point is to make use of the transversal Doppler effect of the returned signal that depends only on the azimuthal component of the flow velocity along the ring-shaped observation beam. We found from a detailed analysis of the experimental method that probing the fluid with LG beams is an effective and simple sensing technique capable to produce accurate estimates of flow vorticity [1].

**Keywords**: Laser Doppler velocimetry; Turbulence; Flow diagnostics; Optical sensing; Atmospheric and oceanic optics; Medical and biological imaging.

## 1. Introduction

Formally, vorticity $\vec{\omega} = \vec{\nabla} \times \vec{U}$ is defined as the curl of the velocity vector $\vec{U}$, which is a measure of angular rotation of a material point about a local position in a flow field and may be regarded as a measure of the local angular velocity of the fluid [2][3]. Typically, vorticity measurement systems are designed to probe one the components of the vorticity vector $\vec{\omega}$ [3]. For simplicity, and without lost of generality, we consider a two-dimensional flow in the $xy$–plane, where $\vec{U} = (U, V, 0)$ and vorticity $\vec{\omega} = (0,0,\omega)$ z–axis component is defined on the basis of velocity derivatives as $\omega = \partial V/\partial x - \partial U/\partial y$.

Several optical methods of vorticity measurement in a flow –Particle Image Velocimetry (PIV) [4][5] and Laser Doppler Velocimetry (LDV) [6][7], among others– attend to determine first the instantaneous velocity components $U$ and $V$, and then differentiate velocity data to yield vorticity field component $\omega$. By whatever means the velocity measurements are estimate, they must be made consecutively with high frequency to attain fine temporal resolution, and simultaneously over several closed space locations from which spatial gradients $\partial V/\partial x$ and $\partial U/\partial y$ can be evaluated using finite difference schemes. However, the fact that these techniques define vorticity in terms of the velocity gradients produces poor accuracy estimates. The accuracy of the calculate vorticity depends on the mean spatial resolution of the velocity sampling –as any real measure needs to consider the length scales $dx$ and $dy$, flow measurements are always integrated over some area and a spatial average vorticity is measured– and the uncertainty error on estimates of velocity differences $dU$ and $dV$ affects vorticity accuracy.

To overcome the shortcomings of finite difference methods for vorticity measurements, here we consider a new optical probing technique that is not implicitly dependent upon velocity measurements and directly observes the local vorticity of fluid elements. It uses LG laser beams to illuminate the flow and obtain, by observing the transversal Doppler effect in the reflected signal [8][9], information about vorticity. It is

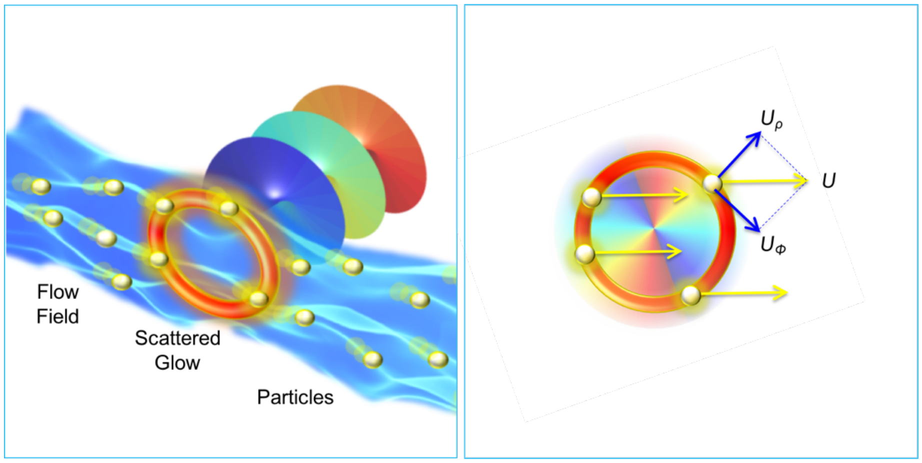


Fig. 1. Measure of vorticity in a flow.

necessary to note that other measurement methods use image techniques to directly observe local flow rotation. Vorticity Optical Probing (VOP) uses Gaussian laser beams to illuminate the passage of probe particles embedded in the flow to obtain, by image analysis, information about their trajectories [10]. Although small probe particles suspended in a flow will react to fluctuations of rotation in the flow, allowing the vorticity of the flow to be probed directly as it moves along a streamline, it is not always possible –or even convenient– to implant probe particles into the fluid whose dynamics needs to be characterized. The technique we propose is akin to laser Doppler anemometry using LG laser beams to illuminate clear flows (see Fig. 1).

The vorticity measurement technique we consider here determines the vorticity $\omega$ directly from the estimation of the transversal Doppler frequency centroid $\langle f_\perp \rangle$ of the signal backscattered by the flow when illuminated by a LG beam with mode number $m$ [1]:

$$\omega = 4\pi/m \ \langle f_\perp \rangle$$

The frequency centroid $\langle f_\perp \rangle$ estimation is typically based on the spectrum of the observed signal. The temporal return backscattered signal –a compound of signals with different frequency Doppler shift triggered by the multiple components of velocity $U_\phi$ along the annular illumination beam, as shown in Fig. 1– can be Fourier transformed to define its frequency spectrum. The characteristic return Doppler spectrum is a histogram of Doppler frequency components describing the spectral content of the returned signal and it can be used to calculate the frequency centroid $\langle f_\perp \rangle$ as the average of the frequencies present in the signal.

## 2. Experiments on the measurement of flow vorticity

We use numerical experiments on the flow vorticity measurement to demonstrate the viability of the proposed method. The use of a realistic signal model illustrates the dependence of the results on the different experimental parameters and allows addressing the problem of vorticity estimation under the supposition of both additive (receiver) and multiplicative (speckle) noises. We assume that the Doppler measurement system uses heterodyne detection –the most straightforward to set up experimentally– where the scattered light is coherently mixed on the receiver with a more intense reference beam, which acts as a local oscillator only.

When a set of independent scatters, moving with velocity $\vec{U}$, passes the ring–like observation region, it generates a burst of optical echoes that contributes to the received optical signal. We use a superposition model for the scattering process that directly gives the complex amplitude of the return signal as the sum

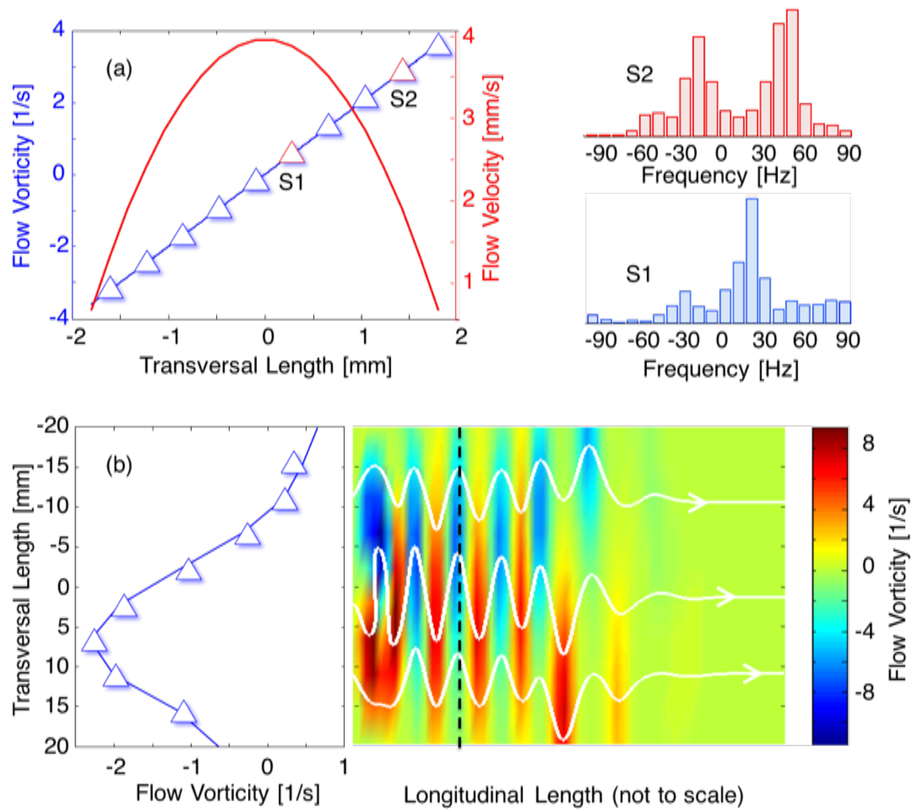


Fig. 2. Numerical experiments on the measurement of flow vorticity

of the fields scattered by all the scatters illuminated by the LG beam. After coherent detection and filtering to remove the carrier frequency and its harmonics, we obtain a detected signal (photocurrent) characterizing the optical echo from the target.

Figure 2 shows the result of our numerical experiments on two different flow patterns. The technique is tested in a steady laminar flow (Fig. 1(a)), in which the flow vorticity is known, and in a complex flow around a circular cylinder immersed in a uniform flow (Fig. 1(b)). We measure vorticity and study the effects of several flow and illumination parameters on the performance of the probing technique. In these experiments, we consider an incident LG laser beam with radial mode number $p = 0$, azimuthal mode number $m = 10$, and beam radius $\omega_0 = 45$ μm. The illuminating beam phase changes from zero to $2\pi$ ten times around the azimuth and the intensity distribution shows a bright ring of radius $\rho_0 = 100$ μm. Typical parameters for a heterodyne receiver are chosen to be signal bandwidth normalized to the sampling frequency $B/F_S = 0.5$, and number of complex samples per estimate $M = 64$. The plots consider a signal return accumulation (spectral accumulation) of $N = 250$.

Figure 2(a) shows the measure of vorticity in a laminar flow. In the numerical experiment, the fluid is flowing along the longitudinal x–axis through a closed channel of radius $R = 2$ mm. The transversal velocity $\vec{U} = (U, 0)$ of flow describes a parabolic profile of velocities along the transversal y–axis that varies from zero at the channel ends to a maximum of $U_0 = 4$ mm/s along the center of the channel. The parabolic profile of velocities $U = U_0[1 - (y/R)^2]$ gives the linear vorticity profile $\omega = 2\, U_0/R^2\, y$. The measurements with LG beams reproduce very closely these expected vorticity values.

In a different numerical experiment, Fig. 2(b) shows vorticity in a complex flow created by the unsteady separation of fluid around a cylindrical object located up stream (don't show in the graph). We estimate the velocity field $\vec{U} = (U, V)$ using a numerical tool for flow simulation. From the numerical velocity field we calculate the resultant z–component of vorticity as $\omega = \partial V/\partial x - \partial U/\partial y$. The flows on opposite sides of the cylindrical object interact in an extended region and produce a regular circulation pattern. The energy of the vortices is ultimately expended by viscosity as they move further down stream and the regular pattern disappears. The velocity field is pictured in the right graph with a set of streamlines that are tangent to the flow velocity vector. The color scale in the same graph gives an idea of the vorticity magnitude. The left plot compares a measure of flow vorticity with LG beams and the corresponding theoretical expectations. In the simulation, the measurement is realized across the flow, down stream from the cylindrical object.

The feasibility of the proposed method to measure flow vorticity was also verified through the experiments. A heterodyne receiver based on a modified Mach-Zehnder interferometer was used for experiments. Using the insight realized by numerical experiments into the problem of vorticity estimation, the operation parameters of the test system were established in order to emulate different types of flows. More details of our analysis, and results on lab experiments on the measurement of flow vorticity, will be presented at the meeting.

## 3. Conclusions

The problem of measuring vorticity in a flow has been confronted. We propose an optical technique that uses LG beams, characterized by ring–like intensity distributions and azimuthal phase variations, to sense rotation at every point in a flow. We develop the theoretical background behind the modeling of optical measurement of vorticity in a flow, identifying the required assumptions and input beam parameters. The spectral properties of the return signal, and the spectrum centroid integral in particular, are fundamental to interpretation of experiments used in flow vorticity monitoring. We verify the working principle of the technique with numerical and lab experiments. By using measurement data, we assess the feasibility of the sensing technique and identify the accuracy of vorticity measurements from return signals affected by target speckle and receiver noise.

## 4. References

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