Improved Analytic Equations for Estimating the Performance of Truncated and Aberrated Gaussian-Beam Coherent Lidar

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Abstract: An approximate analytic equation for modeling the antenna efficiency of coherent lidar systems has been used by the lidar community for many years. This equation assumes perfect untru cated Gaussian beams and as such can lead to significant errors in performance predictions of real lidar systems. The best performance predictions are of course obtained by performing detailed numerical calculations that include all the intensity and wavefront details of the transmitted and back-propagated local oscillator beams, as well as the refractive turbulence profile in the atmosphere, but these calculations can be difficult and time consuming for trading the performance as system parameters are varied. We have developed an improved analytic approximation for calculating the antenna efficiency of Gaussian beam coherent lidar systems that includes the impact of beam quality, truncation, focus, and refractive turbulence. The development of this new equation and comparison of its predictions with those of numerical calculations and the historically used approximate analytic expression will be presented.

Keywords: Coherent Lidar, Modeling, Antenna Efficiency

1. Untruncated Gaussian Beam Lidar Antenna Efficiency

Unaberrated Gaussian beam propagation and antenna efficiency equations including misalignment have been previously described and are only briefly reviewed here due to limited space. In addition, we ignore refractive turbulence in this summary to allow room for more discussion of the impact of truncation and aberrations on antenna efficiency. Impacts of refractive turbulence will be discussed briefly in the presentation.

As described in detail in Reference 1, the untruncated Gaussian beam (UGB) antenna efficiency for arbitrary Gaussian beam sizes and overlap is given by

$$\eta_{a,UGB}(z) = \psi(z) exp\left[-2\left(\frac{\delta r(z)}{\omega_{dB}(z)}\right)^2 \psi(z)\right], \quad (1)$$

$$\text{with,}\ \psi(z) = \left[\frac{\omega_t^2(z)+\omega_b^2(z)}{\omega_{db}^2(z)}\right]^{-1}. \quad (2)$$

In these equations, $\omega_t(z)$ and $\omega_b(z)$ are the e^{-2} intensity radii of the transmit and back propagated local oscillator (BPLO) beams at range z, respectively, $\omega_{db}(z)$ is the diffraction limited spot size of the BPLO given by $\omega_{db}(z) = \lambda z/\pi\omega_{ob}$, where ω_{ob} is the e^{-2} beam waist intensity radius of the BPLO and $\delta r(z)$ is the offset of the two beam centers.

This can also be written as,

$$\psi(z) = \left[\frac{1}{S_t(z)} + \frac{1}{S_b(z)}\right]^{-1}. \quad (3)$$

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where, $S_t(z)$ and $S_b(z)$ are the Strehl ratios of the transmit and BPLO beams, defined as the peak on-axis intensity of the beams relative to that of the untruncated diffraction-limited BPLO beam focused at range z. In other words,

$$S_i(z) = \frac{I_{o,i}(z)}{I_{o,dB}(z)} = \frac{2P}{\pi\omega_i^2(z)} \Big/ \frac{2P}{\pi\omega_{db}^2(z)} = \frac{\omega_{db}^2(z)}{\omega_i^2(z)}\,. \tag{4}$$

The above equations are valid for unmatched Gaussian beam sizes and arbitrary misalignments, but in order to simplify the following discussion on truncated and aberrated beams we assume the beams are co-aligned and matched in size. In this case the above equations simplify to

$$\eta_{a,UGB}(z) = \frac{1}{A_u}\frac{\lambda^2 z^2}{\pi\omega^2(z)} = \frac{S(z)}{2}\,. \tag{5}$$

where $A_u = 2\pi\omega_o^2$ is the effective aperture area of an UGB lidar [1] and where ω_o is the e^{-2} beam waist intensity radius of the transmit beam. Although we assume matched transmit and BPLO beams for simplicity in the rest of this summary, it is straightforward to follow the same development and derive approximate analytic equations for unmatched beam lidar systems.

2. Truncated Gaussian Beam Antenna Efficiency

We next investigate the antenna efficiency of truncated Gaussian beams (TGB). We assume a circular aperture with the incident Gaussian beam centered on the aperture and the truncation level defined by the truncation ratio, $\rho_T = \omega_o/a$, with a being the aperture radius. The far field irradiance profiles of TGBs at moderate truncation ratios have a significant fraction of the diffracted energy in the central diffraction lobe – for example, if $\rho_T = 0.8$, 95.6% of the energy is transmitted through the aperture and 98.3% of that transmitted energy is in the central diffraction lobe. It is therefore reasonable to expect that the UGB antenna efficiency equations can be used to provide a first order estimate for the antenna efficiency of a TGB lidar, especially in the far field at moderate truncation ratios.

For an UGB, the antenna efficiency is a function of the Strehl ratio as shown in Equation 5. Therefore, as a first attempt at deriving a suitable analytic expression for the TGB antenna efficiency, we use what we name the Strehl-based antenna efficiency (SAE) approximation.

The on-axis intensity of a TGB with arbitrary focus and truncation ratio, found by solving the Fresnel integral equation, is

$$I(0,z) = \frac{\frac{2P}{\pi\omega_o^2}}{\left(1-\frac{z}{F}\right)^2+\left(\frac{\lambda z}{\pi\omega_o^2}\right)^2}\left[\left(1+e^{-\frac{2}{\rho_t^2}}\right) - 2e^{-\frac{1}{\rho_t^2}}cos\left[\pi\frac{a^2}{\lambda}\left(\frac{1}{z}-\frac{1}{F}\right)\right]\right]. \tag{6}$$

In terms of the effective Fresnel distance, $\frac{1}{z_{Fe}} = \left|\frac{1}{z}-\frac{1}{F}\right|$, the effective Fresnel number, $N_{Fe} = \frac{a^2}{\lambda z_{Fe}}$, and the magnification, $m = \frac{z}{z_{Fe}}$, this can be written as

$$I(0,z) = \frac{1}{m^2}\frac{I(0,0)}{1+\left(\frac{1}{\rho_t}\right)^4\left(\frac{1}{\pi N_{Fe}}\right)^2}\left[\left(1+e^{-2\left(\frac{1}{\rho_t}\right)^2}\right) - 2e^{-\left(\frac{1}{\rho_t}\right)^2}cos[\pi N_{Fe}]\right]\,. \tag{7}$$

Thus, the on-axis intensity is only a function of the truncation ratio, the effective Fresnel number, and a magnification factor, which depends on the focus conditions. In general, the 2-dimensional intensity patterns of the diffracted beam at any range are also only a function these parameters. Example, radial intensity profiles as a function of effective Fresnel number are shown in Figure 1 for a truncation ratio of 0.8. Also shown is the intensity profile of the untruncated Gaussian beam, and the Gaussian beam fit determined using our CSAE model, with proper irradiance scaling for reference.

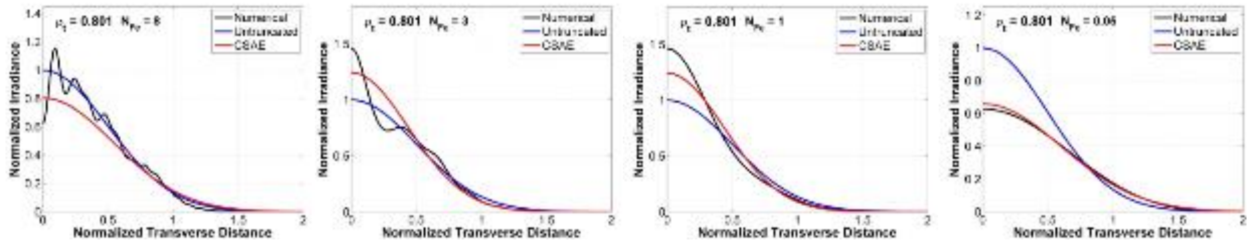

Figure 1. Example normalized irradiance profiles for a TGB vs. Fresnel number. The UGB and the CSAE Guassian beam (described later) are also shown with correct relative irradiance for reference.

The truncated and aberrated irradiance profiles presented throughout this paper are numerically calculated using a diverging grid FFT beam propagator algorithm based on the Sziklas-Siegman algorithm [2,3]. The diverging grid approach permits one to propagate finite and phase modulated beams over long distances while maintaining sufficient grid sampling.

The numerically calculated antenna efficiencies shown later in this paper are computed from the target plane normalized irradiance overlap integral of the transmit and BPLO fields as

$$\eta_a(z) = \eta_{Tx}\eta_{Tb}\frac{\lambda^2 z^2}{A_r}\iint I_{nx}(x,y,z)\, I_{nb}(x,y,z)\,dx\,dy\,, \tag{8}$$

where η_{Tx} and η_{Tb} are the truncation efficiencies of the transmit and BPLO beams, respectively, and I_{nx} and I_{nb} are, respectively, the normalized to unit power two dimensional target plane irradiance distributions of the transmit and BPLO fields. As noted above, for this paper, we assumes matched transmit and BPLO beams, so $\eta_{Tx} = \eta_{Tb} = \eta_T$.

Using Equation 6, we find that the Strehl ratio is

$$S(z) = \frac{I(0,z)}{2P/\pi\omega_d^2(z)} = \frac{\omega_d^2(z)}{\omega^2(z)}\left[\left(1 + e^{-\frac{2}{\rho_t^2}}\right) - 2e^{-\frac{1}{\rho_t^2}}\cos\left[\pi\frac{a^2}{\lambda}\left(\frac{1}{z} - \frac{1}{F}\right)\right]\right], \tag{9}$$

$$\text{with,}\;\; \omega^2(z) = \omega_o^2\left[\left(1 - \frac{z}{F}\right)^2 + \left(\frac{\lambda z}{\pi\omega_o^2}\right)^2\right]. \tag{10}$$

To convert from the UGB beam antenna efficiency expression to the TGB SAE approximation we account for the different effective aperture areas and the truncation losses using

$$\eta_{a,S}(z) \approx f_c\frac{A_u}{A_r}\eta_T^2\eta_{a,UGB}(z) \approx \eta_T\rho_T^2 f_c S(z) = \eta_T^2\frac{(\lambda z)^2}{A_r}\frac{1}{\pi\omega_{eff,S}^2(z)}\,, \tag{11}$$

where $\frac{A_u}{A_r} = \frac{2\pi(\rho_T a)^2}{\pi a^2} = 2\rho_T^2$, $\omega_{eff,S}^2(z) = \frac{\eta_T}{f_c S(z)}\omega_d^2(z)$ is the effective Gaussian beam size, $\omega_d(z) = \lambda z/\pi\omega_o$, $\eta_T = 1 - \exp(-\frac{2}{\rho_t^2})$ is the truncation efficiency, and f_c is a correction factor to allow an improved fit to the numerically calculated far-field antenna efficiency.

In the far field, when $N_{Fe} \ll 1$, the SAE model works well, but a small correction factor, f_c, is needed to account for the fact that the far field beams are not exact Gaussians. The antenna efficiency vs truncation ratio for a matched beam coherent lidar is shown in the left panel of Figure 2. Note the good agreement with the numerical prediction over most truncation ratio values.

The required correction factor as a function of the truncation ratio is shown in the right panel of Figure 2. The correction factor is small varying between 0.95 and 1.05 over truncation ratios from 0 to 2. Below truncation ratio of 0.5 the beams are UGBs, and SAE model is exact.

Upon careful comparison of the antenna efficiency predicted by the SAE model with numerically calculated values, we found that the SEA model overestimates the antenna efficiency in the intermediate field (with N_{Fe} between approximately 0.25 and 1.5). Upon examination of the diffraction patterns in this

Fresnel number range, this is understood as due to the on-axis irradiance peak occurring over this range (near $N_{Fe} = 1$) resulting in a Gaussian beam fit with a waist size, $\omega_{eff,S}$, that is too small, which in turn results in a predicted antenna efficiency that is too high.

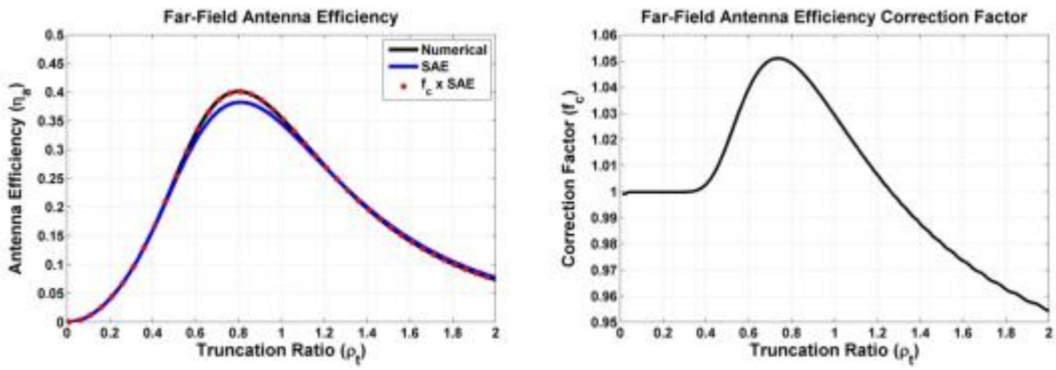

Figure 2. Left Panel: Blue curve - predicted antenna efficiency using the SAE model. Black curve- numerical calculation. Red dashed – SAE model using correction factor shown in the right panel. Right Panel: Required correction factor, f_c, with truncation ratio.

This is illustrated in Figure 3 for two focus conditions of 500 m (top row) and infinity (bottom row)) and 4 truncation ratios, 0.55, 0.8, 1.2, and 2.0. The antenna efficiency of three different models is compared to the numerically calculated antenna efficiency (see Figure caption). At a truncation ratio of 0.55 all the models are good since the beams are essentially untruncated. Note that the scaled (to the far field antenna efficiency) UGB model significantly underestimates the antenna efficiency over much of the range and underestimates the depth of focus, especially at higher truncation ratios. The SAE model (which now includes the f_c factor) does better than the scaled UGB model, but as discussed above it overestimates the antenna efficiency in the intermediate field. The corrected Strehl antenna efficiency (CSAE) analytic model, which we describe below, fits the antenna efficiency very well.

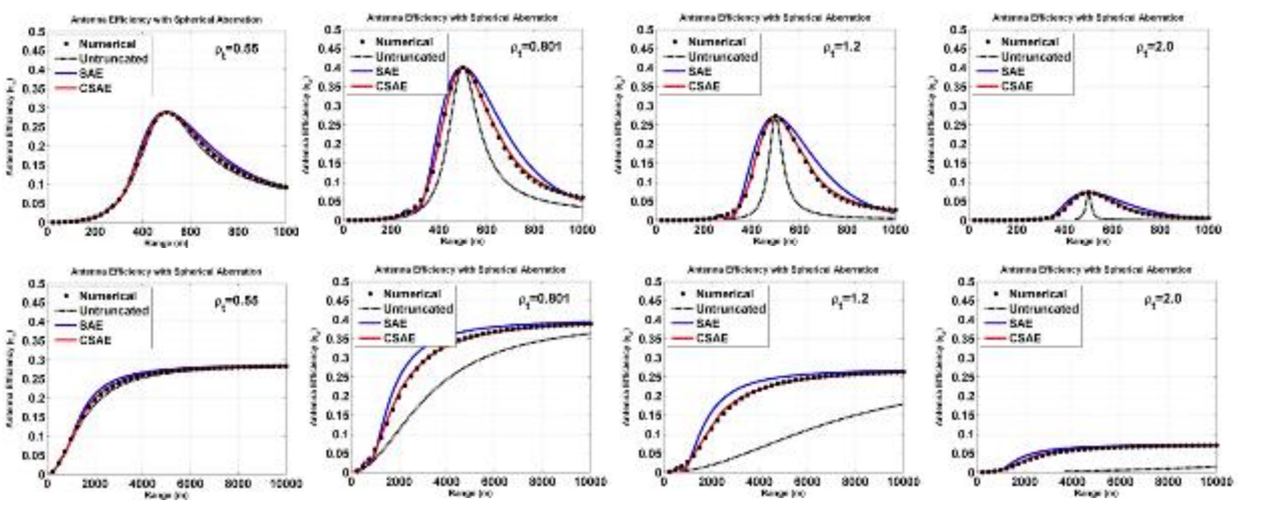

Figure 3. Comparison of numerically calculated antenna efficiency (dots) with: Black Dashed - the UGB model scaled to match the numerically calculated far-field antenna efficiency. Blue- the SAE model with the correction factor shown in Figure 3 applied. Red – Red the CSAE model.

We took into account the above noted behavior in the different effective Fresnel number ranges and constructed the CSAE analytic model to better optimize the performance across the entire range of Fresnel numbers.

First, we define two regimes in terms of the effective Fresnel distance which meet at $z_{Fe} = z_b$, given by

$$z_b = z_R \sqrt{\frac{b - 1/{M^2}^2}{{M^2}^2 - b}} \ , \tag{12}$$

where $z_R = \frac{\pi \omega_0^2}{\lambda}$, $b = \frac{1-exp\left(-\frac{2}{\rho_T^2}\right)}{1+exp\left(-\frac{2}{\rho_T^2}\right)}$, and M^2 is the beam spreading factor given by

$$M^2 = \sqrt{\frac{\eta_{Tx}}{f_c S(F)}}. \tag{13}$$

where $S(F) = \left(1 - e^{-\frac{1}{\rho_t^2}}\right)^2$ is the far field Strehl ratio (at the focus),

In the regime $z_{Fe} \leq z_b$

$$\omega_{eff,l}^2 = b\omega^2 \ . \tag{14}$$

with ω from Equation 10.

In the regime $z_{Fe} > z_b$

$$\omega_{eff,g}^2 = \omega_0^2 \left[\frac{1}{{M^2}^2}\left(1 - \frac{z}{f}\right)^2 + {M^2}^2 \left(\frac{z}{z_R}\right)^2\right]. \tag{15}$$

The refined effective beam size to use in Eq. 11 is then given by the average of the squares as

$$\omega_{eff}^2 = \frac{\omega_{eff,s}^2 + \omega_{eff,l}^2}{2} \quad for \quad z_{Fe} \leq z_b \tag{16}$$

and

$$\omega_{eff}^2 = \frac{\omega_{eff,s}^2 + \omega_{eff,g}^2}{2} \quad for \quad z_{Fe} > z_b \tag{17}$$

3. Aberrated and Truncated Beams

We have also developed an analytic expression that approximates the loss due to aberrations. The analytic Strehl expression is used to find the impact of the defocus aberration, which in terms of the rms aberration over the full aperture is

$$\eta_{a,ATGB} \approx f_c \eta_T \rho_t^2 S = \frac{f_c \rho_t^2 \left(1 - e^{-\frac{2}{\rho_t^2}}\right)}{\left[1 + \left(2\pi\sqrt{12}\, \rho_t^2 \sigma_{df}\right)^2\right]} \left[\left(1 + e^{-\frac{2}{\rho_t^2}}\right) - 2e^{-\frac{1}{\rho_t^2}} cos\left[2\pi\sqrt{12}\, \sigma_{df}\right]\right] \tag{18}$$

We find that with a slight modification to this expression we can approximate the far field antenna efficiency for specific primary aberrations. In each case we replace $\sqrt{12}$ in the numerator and denominator of the above expression with the following: for defocus $\sqrt{18\sqrt{\rho_t}}$, for spherical less defocus $\sqrt{\frac{24}{\rho_t}}$, and for astigmatism less defocus $\sqrt{12\rho_t}$. The numerical calculated antenna efficiency as a function of the rms wavefront aberration for these primary aberration types as well as coma less tilt, and for random aberrations, is shown in Figure 4. This is similar to past work [5,6] but with the primary difference that we remove defocus and tilt to minimize the rms phase over the full aperture.

For truncation ratios of 0.801 and larger (i.e., for fairly well filled aperture conditions) the degradation of the antenna efficiency with wavefront aberration predicted by these analytic expressions agrees relatively well with the simple Strehl expression given in Wyant [7] for a uniformly filled aperture, $S \approx e^{-(2\pi\sigma)^2}$, scaled to the maximum far field antenna efficiency. At lower truncation ratios, where the aperture is less well filled, the beams see less of the aberration of the full aperture and the degradation is less.

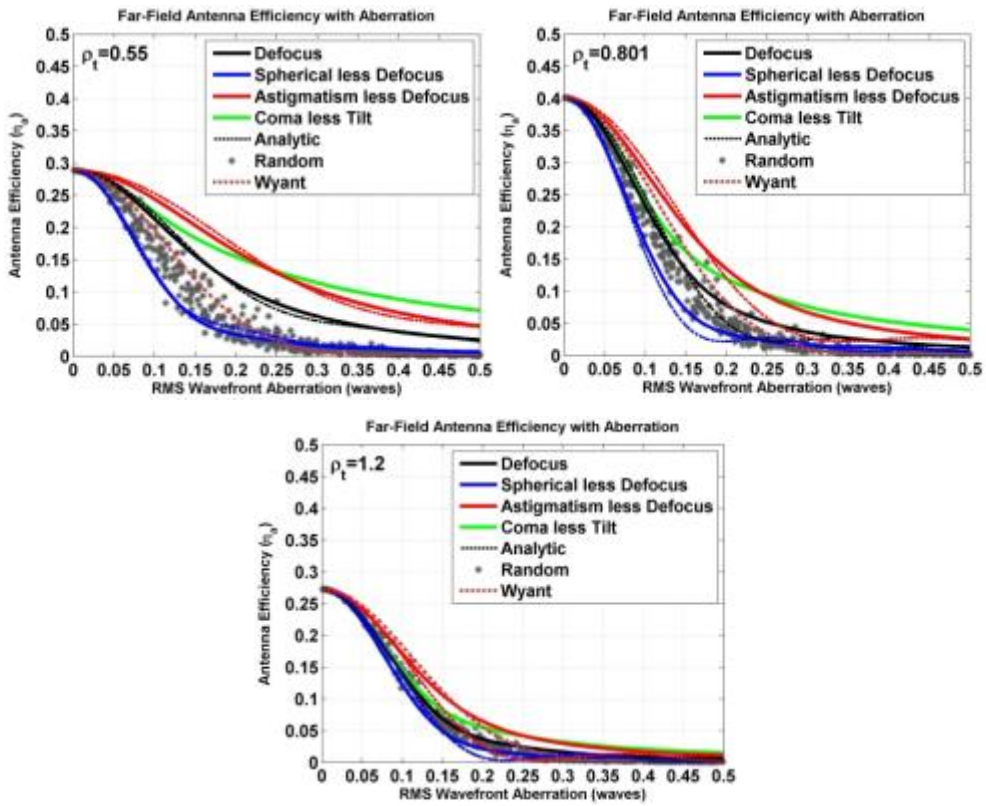

Figure 4. Comparison of antenna efficiency vs. rms wavefront aberration level for: solid curves - numerically calculated using primary aberrations, light dots –numerically calculated using random aberrations, dashed - analytic model prediction.

4. Conclusions

An improved analytic model for estimating the antenna efficiency of truncated and aberrated Gaussian beam coherent lidar systems has been developed. We are continuing to improve the model and more details will be given in the presentation.

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