Velocity Rate Uncertainty and Dark Count Rate Influences on Time of Flight Measurement

Patrick Perkins

Lockheed Martin Advanced Technology Center

patrick.perkins@lmco.com

Statement of Problem

- Range to the target and the range rate of the target are known with some precision
- Lack of precise knowledge requires a search algorithm which looks over the uncertainty in range and range rate
- Low probability of detecting a true target return requires one use a correlation algorithm (usual maximum entropy search)
- The number of possible vectors can be very large
- This is a classic binomial integration problem(require M returns along a range and range rate vector as a condition for detection)
- Use slowly varying algebraic derivation to derive receiver operating characteristic (ROC) over a wide parameter space

Definition of Terms

- true range {R}; time delay {t_{est}} is {2xR/c} = time of flight {TOF}
- time of flight: time for pulse from XCVR to be transmitted and received
- range estimate {R_{est}}; {t_{est}} = {$2xR_{est}/c$}
- laser pulse width {t_{pulse}}
- set of initial range bins {R_{bins}}
- dark count rate {DCR} in count/sec
- white count rate (WCR) in counts/sec

  (photon count rate x probability of single pulse detection ≈ 0.25;

  100 photons/sec = WCR of 25 counts/sec Poisson average)
- energy required {E_1}, on average, to yield a single counted event

Link Budget Example; E^1_{Joule}

- Range = 15000 km = 15 Mm; (speed of light = 300 Mm/s)
- TOF = 100 ms >> $\{t_{pulse}\}$ (<200 ns pulse stretched fiber laser)
- ε_{Joule} = link return in units of counts/J; use dB in m^2 for convenience
  - Range = 15 Mm; R^{-2} = -143.5 dB
  - Target cross section, Ω $\{m^2\} = \rho$ x Area = 0.5 m^2 = -3 dB
  - Full angle Gaussian beam divergence, Θ_{half} = 1.5 μR; power gain of 115 dB
  - Receiver area, $(A_{rcvr}\}$ = 1 m^2 = 0 dB
  - System thruput $\{\eta\}$ = -12 dB, includes probability of detection
  - Photons/Joule = $\Phi_{Joule} = 5.03 \times 10^{18} \times \lambda$ [microns] = 187 dB at 1 micron
  - $\eta\ \Omega\ A_{rcvr}\ \{\Pi/2\ \Theta_{half}{}^2\}\ R^{-4}\ \Phi_{joule}$ = { -12 -3 + 0 + 115 – 287 + 187 = 0 }
- Return 1 count for 1 Joule of output energy, on average

  E^1_{Joule} = 1 (How convenient!)

Simple Model Without Binomial Integration

- Pick a value of total WC's to determine the ROC; use $\{N_{true} = 4\}$
- Nominal laser output 10 mJ at 10 kHz, pulse stretched fiber laser
- Need 400 pulses to output 4J
- $K_{int,}$ number of pulses, $= (E^{ROC}_{int}/P^{avg}_{out}) \times PRF = 4/100 \times 10{,}000 = 400$

- Consider fixed target
  - No increase in TOF window due to range rate "motion" in time
  - Range uncertainty = $R_P - R_V = R_{P\text{-}V}$
  - Time uncertainty, $\{\tau_{P\text{-}V}\} = 2 \times R_{P\text{-}V}/C$
  - Total time gate $\{T = k_{int} \times \tau_{P\text{-}V}\}$
  - For 15 km range uncertainty , $\tau_{P\text{-}V} = 100\ \mu s$

- $SNR_{simple} \cong N_{true}/DC^{½} = N_{true}/(DCR \times k_{int} \times \tau_{P\text{-}V})^{½}$;

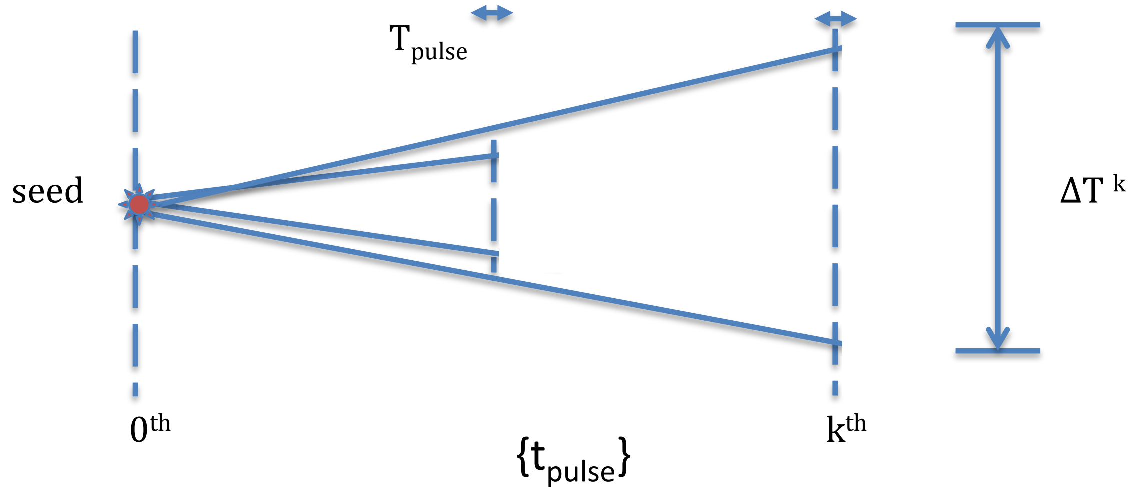
  $= Ntrue \times [P_{out}/(E^{ROC}_{int} \times DCR \times \tau_{P\text{-}V} \times PRF]^{½}$;

  $= [P_{out}/(E^{1}_{int} \times DCR \times \tau_{P\text{-}V} \times PRF]^{½}$

  Usual inverse root dependence in DCR and PRF

Does not show vector field increase in noise due to range and range rate uncertainty!

Adding Range Rate Uncertainty



A seed is defined as a range uncertainty element: $C \times t_{pulse}$. Total number of seeds covers total range uncertainty. This is the usual multi hypothesis type of algorithm.

$\{k_{int}\}$ is still set by the ROC; however, the time window must grow to accommodate the change in range. For a given number of pulses, the total time growth of the acceptance window is $\{\Delta T^{\ k} = k_{int}/PRF \times 2\delta_V /C\}$

As the required output energy is reduced, the number of pulses integrated and the range rate uncertainty time window are linearly reduced. Hence, there is a square dependence on the value of $\{E^{\ 1}_{int}\}$.

Estimating Increased Window Time

- $\Delta T^{k} = (E^{ROC}_{int} / P_{out}) \times (2\delta_V / C)$;
- $N^{k}_{lines} = (E^{ROC}_{int} / P_{out}) \times (2\delta_V / C) / \tau_{pulse}$ {divide by the pulse width}.
- The number of points per line and thus the total time window for all points along all lines is as follows for a given seed:
- $N^{points}_{line} = k_{int} = (E^{ROC}_{int} / P_{out}) \times PRF$;
- $N^{points}_{seed} = (E^{ROC}_{int} / P_{out})^2 \times PRF \times (2\delta_V / C) / \tau_{pulse}$;
- $\Delta T^{total}_{seed} = (E^{ROC}_{int} / P_{out})^2 (PRF \times 2\delta_V / C)$ [units of s] .

Note: each seed has the same dependency

Simple Range Rate Dependencies

Note that as we reduce the number of lines as we increase the laser pulse width or the sample time. We now calculate the noise at each "point" along the projected velocity vector which is simply {DCR x τ_{pulse}}. Thus, the total noise in the velocity estimator to first order does not have pulse width dependence. The number of points is reduced as you increase the laser pulse width but the time gate is linearly increased as well. The result is that the total number of DC's is pulse width independent.

$$DC^{points}_{seed} = (E^{ROC}int / P_{out})^2 \times [DCR \times PRF \times 2\delta_V / C].$$

This is a unit less quantity; the first term has the units of S^2; the second term the units of S^{-2}. Let's make a sample calculation when we expect a value of unity for the variance. The first term is 4J required with an 100W output or {1/625 s^2}. We estimate the DCR and PRF as 5 kHz and 10 kHz. Thus, prior to evaluating the "Doppler" term, we have a numerical estimate of 80,000 noise counts x {$2\delta_V/C$}.

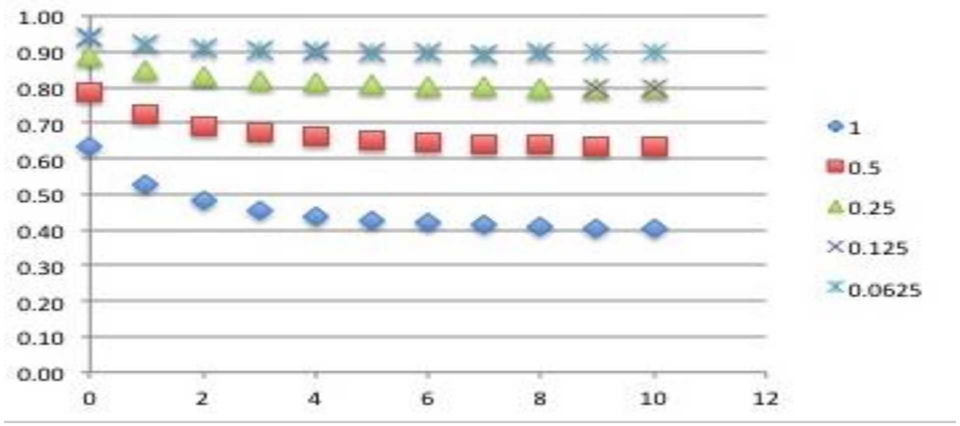
Use a value of 125 ppm for the velocity uncertainty and we would expect 10 noise counts per seed. Increasing the laser pulse width inversely reduces the number of range bin seeds which helps considerably in the tracking process. It does not change the velocity search along a given vector but does reduce the total number of vectors.

If we consider the variance as the root of the number of DC's, we recover the root dependencies.

Probability and ROC

- How many DC's along a single line?
- 400 pulses x 200 nS x 1000 Hz (DCR) = 0.08 DC's per kHz
- Sufficiently small we can approximate Poissonian probability function
- P_{FA} is the Poisson probability {DC< N_{true}}
- $P_{\text{single track}}$ [DC's < N_{true}] < $DC^{Ntrue} / N_{true}!$

The following graph shows the validity of this expansion by comparing the simple exponential and factorial model with the actual Poisson distribution as the expectation value is decreased in factors of 2 from unity. Consider this an upper bound.



Driving Toward Binomial Integration

The total probability that an erroneous vector will be named is the single track probability multiplied by the total number of possible vectors or

$$P_{error} < [(k_{int}\, \tau_{pulse}\, DCR)^{Ntrue} / N_{true}!]\; [(E^{ROC}_{int}/P_{out})(2\delta_V/C) / \tau_{pulse}].$$

We inject the number of WC's by defining [$E^{ROC}_{int} \cong WC\, E^{1}_{int}$] where the superscript {1} denotes the amount of energy required to return, on average, unity WC's. Recall that {$k_{int} = (E^{ROC}_{int}/P_{out}) \times PRF$}, using n instead of N_{true},

$$P^{n}_{error} < (2\delta_V/C)\, PRF^{n}\, WC^{n+1}\, (E^{1}_{int}/P_{out})^{n+1}\, DCR^{n}\, \tau_{pulse}^{n-1} / n! \;\;;$$
$$P^{n}_{error} < (2\delta_V/C)\, \tau_{pulse}^{n-1}\, (PRF\, DCR)^{n}\, WC^{n+1}\, (E^{1}_{int}/P_{out})^{n+1} / n! \;\;.$$

Using values of 5 kHz for PRF, 1000 Hz for DCR, as well as (1/100 sec) for the energy/power ratio, a value of 4 for WC's, and 10^{-4} for the relative velocity uncertainty,

$$P^{1}_{error} < (10^{-4})\, (200 \times 10^{-9})^{1-1}\, (5 \times 10^{3} \times 1000)^{1} \times 4^{1+1} \times (1/100)^{2} = 0.8$$

which has the units of [$s^{n-1}\, s^{-2n}\, s^{n+1}$] and is correctly dimensionless.

We could not expect the algorithm to select the correct velocity vector with that much noise. At this point, we examine a 90% detection probability for the system and use binomial integration, requiring M detects out of N WC's for a true detection.

WC's for a given ROC

- We can readily generate the number of WC's required for a given selection criterion. The ROC is determined in the following way:
- Set the criterion. If you select {3}, you require three detections that fall on a straight line in velocity space where straight means that detections are within the laser pulse window.
- Recursively solve the Poisson probability function such that the probability of detecting Ntrue from WC is greater than your threshold probability of 5% or as you choose.
- Solve the error equation for the DCR that also yields an error probability of 5% or your threshold level of error.

WC Expectation Values

Criterion \ Missed %	2.5	5	10	20	40
3	7.20	6.3	5.30	4.30	3.10
4	8.75	7.75	6.65	5.50	4.20
5	10.25	9.15	8.00	6.70	5.25
6	11.70	10.5	9.25	7.90	6.30
7	13.05	11.85	10.50	9.10	7.35

Example Calculation

We are using a PRF of 10 kHz, a pulse width of 200 nS and require 3 points on the velocity track. With a DCR of 1000 Hz, the probability of a DC induced error is 5%.

$(E1/P_{out})$	1.25E-02	[s]
WC	6.3	
$(2\delta_V/C)$	2.00E-04	
PRF	1.00E+04	[s-1]
DCR	1.00E+03	[s-1]
τ_{pulse}	2.00E-07	[s]
time unity return	1.25E-02	[s]
Ntrue	3	
Perror	5.1%	
kint	125.0	

WC	Ntrue≥	DCR
6.3	3	1000
7.75	4	1720
9.15	5	2420
10.5	6	3070
11.85	7	3640
13.15	8	4160

If we use a 4-criterion, we need to integrate for (7.75/6.3) = 125% longer. We can tolerate a DCR of 1720 Hz. The table summarizes a ROC table generated from these example values assuming a 5% error value. With a little more algebra, we can solve for DCR as a direct function of the ROC parameters for a given probability. Alternatively, we can generate a ROC where we use WC as an adjustable parameter, DCR fixed, and we generate error probabilities for a different value of N_{true} for each family of curves.

Scaling with PRF

It is interesting to determine how this scales with laser PRF. If the laser output power is peak power constrained but not gain constrained, the error equation is simplified. The product of [PRF x $\tau_{pulse} \cong Y_{laser}$], which is dimensionless, can be considered constant. As an example, {Y_{laser} = 10 kHz x 200 nS = 2 x 10^{-3}}. We rewrite the governing error equation, peak power limited, as

$$P_{DC}{}^{n}{}_{error} < (2\delta_V/C)\, WC^{n+1} (E^1{}_{int}/P_{out})^{n+1}\, Y_{laser}{}^{n-1}\, DCR^n\, PRF/n!$$

This quantity is dimensionless: [$s^{n+1}\, s^{-n}\, s^{-1}$]. From this, it is clear for the peak power constrained laser pulse energy assumption, the probability of detecting a spurious velocity track is proportional to PRF but not a function of laser pulse width. This is for a single seed. We have to multiply this by the number of seeds to look at the entire problem.

Thus,

$$\tau_{pulse} \cong Y_{laser}/PRF$$

$$N_{seeds} = \{2\,\delta_R/C\} / \tau_{pulse} = PRF\, \{2\,\delta_R/C\} / Y_{laser}.$$

The net result is a PRF^2 dependency in the line error. Noting the nearly exponential nature of the error, if you have 100 seeds, you would need natural log (100) = 4.8 more WC's to handle the seed field, any which could contribute to the error signal.

This process converges most quickly when the PRF is as low as possible within the laser constraint equation.

Approximating the ROC

We can approximate [WC] as a function of the decision criterion, n ranging from 3 to 8 as :

$$\text{WC } (P_{\underline{det}} < 5\%) \quad \approx \quad \log_2 (n^4) + (n-3)^3/128 \quad .$$

One can generate readily a ROC of high utility using a simple approximation fit. Log's work quite well as you would expect from other expansions of the binomial and Poisson distributions.

	5% ROC		
WC	Ntrue≥	approx	relative error
6.30	3	6.34	0.6%
7.75	4	8.01	3.3%
9.15	5	9.35	2.2%
10.50	6	10.55	0.5%
11.85	7	11.73	-1.0%
13.15	8	12.98	-1.3%