Improved Analytic Equations for Estimating the Performance of Truncated and Aberrated Gaussian-Beam Coherent Lidar

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Outline

• Lidar CNR Equation and Antenna Efficiency Review
• Untruncated Gaussian Beam Coherent Lidar Antenna Efficiency
• Aberrated Untruncated Gaussian Beams
• Truncated Gaussian Beams - Motivation for Improved Analytic Equations
• Strehl-ratio based Analytic Equations
• Impact of Aberrations
• Summary
Lidar CNR and Antenna Efficiency

- Short Pulse Lidar CNR
  \[ CNR(z) = \eta_a(z)\eta_{eo}T^2 \frac{E}{hvB} \frac{c\beta A_r}{2z^2} \]
  \[ \eta_{eo} = \eta_t\eta_r\eta_q\eta_{bc}\eta_{sn} \]

- Coherent Lidar Antenna Efficiency
  \[ \eta_a(z) = \eta_{Tt}\eta_h(z) = \eta_{Tt}\eta_{Tb} \frac{\lambda^2z^2}{A_r} \iint I_{nt}(x, y, z) I_{nb}(x, y, z) dx dy \]
  - Truncation Ratio of the Transmit and BPLO beams are \( \eta_{Tt} \) and \( \eta_{Tb} \) respectively.
  - Intensities are normalized wrt the total power in the target plane

- Effective Aperture Area = \( \eta_aA_r \)

- Antenna Efficiency Sometime called Signal Reduction Factor
Untruncated Gaussian Beams

• Assuming Untruncated Gaussian Beams (UGB), Propagation Equation

\[ \omega^2(z) \approx \omega_o^2 \left(1 - \frac{Z}{F}\right)^2 + \left(\frac{\lambda Z}{\pi \omega_o}\right)^2 \]

• Antenna Eff. Integral with arbitrary beam sizes & alignment results in UGB Effective Receiver Area of

\[ A_{r,UGB} = 2\pi \omega_{ob}^2 \]

• and UGB Antenna Efficiency [1]

\[ \eta_{a,UGB}(z) = \psi(z) \exp \left[-2 \left(\frac{\delta r(z)}{\omega_{dB}(z)}\right)^2 \psi(z)\right] \]

with \( \psi(z) = \frac{\omega_{db}^2}{\omega_t^2(z) + \omega_b^2(z)} \) and \( \omega_{db}(z) = \frac{\lambda Z}{\pi \omega_{ob}} \),

If Pupil Plane beam sizes are matched

\[ \psi(z) = \left[2 + 2(1 - z/F)^2 \left(\frac{Z_R}{Z}\right)^2\right]^{-1}, \text{with } Z_R = \frac{\pi \omega_o^2}{\lambda} \]

If Pupil Plane beam offsets are zero

\[ \frac{\delta r(z)}{\omega_{dB}(z)} = \frac{\delta \theta}{\theta_{dB}}, \quad \text{with } \theta_{dB} = \frac{\lambda}{\pi \omega_{ob}} \]

Aberrated Untruncated Gaussian Beams

• Beam propagation modified by beam quality, or far-field beam spreading factor, so far-field divergence increases by $M^2$ [2,3],

$$\theta_{d,M^2} = M^2 \theta_d = M^2 \frac{\lambda}{\pi \omega_0}$$

• Beam propagation equation

$$\omega^2(z) \approx \omega_0^2 \left(1 - \frac{z}{F}\right)^2 + \left(M^2 \frac{\lambda z}{\pi \omega_0} \right)^2 + \frac{1}{g_m} \left(\frac{\lambda R}{\pi \rho_o(R)} \right)^2$$

• Lidar antenna efficiency

$$\eta_{a,AUGB}(z) \approx \psi(z) \exp \left[-2 \left(\frac{\delta r(z)}{\omega_{dB}(z)}\right)^2 \psi(z)\right]$$

$$\psi(z) = \frac{\omega_{dB}^2}{\omega_t^2(z) + \omega_b^2(z)}$$

If Pupil Plane beam sizes are matched

$$\psi(z) = \left[ M_b^2 + M_t^2 + 2 \left(1 - z/F\right)^2 \left(\frac{Z_R}{z}\right)^2 \right]^{-1}$$

If Pupil Plane beam offsets are zero

$$\frac{\delta r(z)}{\omega_{dB}(z)} = \frac{\delta \theta}{\theta_{db}}$$, with $\theta_{db} = \frac{\lambda}{\pi \omega_{ob}}$


[3] J.P. Cariou, et. al. “Laser source requirements for coherent lidars based on fiber technology,” C.R Physique & (2006) previously presented a related far-field-only version of this equation with no range or BPLO beam size dependence
Written in Terms Of Strehl Ratio

- **Strehl Ratio** is defined as ratio of On-Axis Beam Intensity to the On-Axis Intensity possible with a focused Diffraction-Limited Gaussian beam.

- **For Gaussian Beams**
  \[
  \eta_{A,\text{UGB}}(z) \approx \psi(z) \exp \left[ -2 \left( \frac{\delta r(z)}{\omega_{dB}(z)} \right)^2 \psi(z) \right]
  \]
  \[
  \psi(z) = \frac{\omega_{db}^2}{\omega_t^2(z) + \omega_b^2(z)} = \left[ \frac{1}{S_t(z)} + \frac{1}{S_b(z)} \right]^{-1}
  \]

- **In the Far Field**
  \[
  M_i^2 = \frac{1}{\sqrt{S_i(F)}}
  \]

- **To Simplify Expressions, Assume Matched Transmit and BPLO Beams, Can easily add unmatched beams back in at end following Development**
  \[
  \psi(z) = \frac{S}{2}
  \]
Behavior of UGB Antenna Efficiency

- Most Publications do not Properly Include Range Dependence of Misalignment Term
- Not Sensitive to Misalignment in Near Field
- ”Peaking” at Focal Range Depends on Misalignment

\[ F = \infty \]
\[ M^2 = 1 \]

Assumes UGBs with \( \omega_o = 0.4 \) m and \( \lambda = 1565 \text{ nm} \)
Behavior of Aberrated UGB Antenna Efficiency

- Parametric in $M_t^2$ with $M_b^2 = 1$
- Various misalignment values

\[
\frac{\delta \theta}{\theta_d} = 0 \\
F = 2 \text{ km}
\]

\[
\frac{\delta \theta}{\theta_d} = 0.5 \\
F = 2 \text{ km}
\]

\[
\frac{\delta \theta}{\theta_d} = 1 \\
F = 2 \text{ km}
\]
UGB Model is Not Accurate Approx. for Truncated Gaussian Beams

- Examination began while trying to improve accuracy of Lidar calibration and Lidar system efficiency measurements.
- System design trades much easier/faster using analytic models.
- But poor agreement between numerically calculated TGB antenna efficiency and UGB analytic model results in significant errors:
  - UGB model significantly underestimates antenna efficiency (lidar CNR) over much of the range (left figure and center figure).
  - Significant underestimation of depth of focus (center figure).
  - Overestimates misalignment impact (right figure).
• Even Worse Agreement at Higher Truncation Ratios

• New Analytic Model Objectives
  – Significantly improved fit over all ranges and for all truncation values with minimal fitting parameters
  – Include Impacts of Optical Aberrations
  – Include Impact of Refractive Turbulence
Truncated Gaussian Beam Strehl-based Antenna Efficiency (SAE) Approximation

- Translation of UGB Antenna Efficiency to Real Truncated Beam Aperture
  \[ \eta_{a,TGB}(z) \approx f_c \frac{A_u}{A_r} \eta_T^2 \eta_{a,UGB}(z) \quad \text{with} \quad \frac{A_u}{A_r} = \frac{2\pi(\rho_T a)^2}{\pi a^2} = 2\rho_T^2 \quad \text{and} \quad \eta_T = 1 - \exp\left(-\frac{2}{\rho_T^2}\right) \]

- Using The UGB Antenna Efficiency Strehl-ratio relation
  \[ \eta_{a,UGB}(z) = \frac{1}{A_u} \frac{\lambda^2 z^2}{\pi \omega^2(z)} = \frac{S(z)}{2} \]

- Yielding the Strehl-based Antenna Efficiency (SAE) Approximation
  \[ \eta_{a,TGB}(z) \approx \eta_T \rho_T^2 f_c S(z) = \eta_T^2 \frac{(\lambda z)^2}{A_r} \frac{1}{\pi \omega_{eff,S}^2(z)} \]

  \[ \omega_{eff,S}^2(z) = \frac{\eta_T}{f_c S(z)} \omega_d^2(z) \quad \omega_d^2(z) = \frac{\lambda z}{\pi \omega_o} \]
Fresnel Diffraction Patterns

\[
\rho_T = 0.8
\]

\(N_{Fe} = 8\)  
Near Field

\(N_{Fe} = 3\)  
Near Field

\(N_{Fe} = 1\)  
Intermediate Field

\(N_{Fe} = 0.05\)  
Far Field
On-axis Intensity of Truncated Gaussian Beams

- Fresnel Equation can be Solved Analytically for On-Axis Intensity
- Strehl Ratio, defined as on axis intensity compared to that of the DL of the untruncated Gaussian beam

\[ S(z) = \frac{I(0, z)}{2P/\pi \omega_d^2(z)} = \frac{\omega_d^2(z)}{\omega^2(z)} \left[ (1 + e^{\frac{-2}{\rho_T}}) - 2e^{\frac{-1}{\rho_T}} \cos \left( \pi \frac{a^2}{\lambda} \left( \frac{1}{z} - \frac{1}{F} \right) \right) \right] \]

with,

\[ \omega^2(z) = \omega_0^2 \left[ \left( 1 - \frac{Z}{F} \right)^2 + \left( \frac{\lambda Z}{\pi \omega_0^2} \right)^2 \right] \]
Near Field & Far Field Behavior

\[ \eta_{ref,\text{TBG}}(z) \approx \eta_T \rho_T^2 f_c S(z) \]

\[ S(z) = \frac{I(0, z)}{2P/\pi \omega_d^2(z)} = \frac{\omega_d^2(z)}{\omega^2(z)} \left[ \left( 1 + e^{-2} \right) - 2e^{-\frac{1}{\rho_T^2}} \cos \left( \frac{\pi a^2}{\lambda} \left( \frac{1}{z} - \frac{1}{F} \right) \right) \right] \]

- Near Field – At very high Fresnel Numbers, Lidar Antenna Efficiency does not respond to fast modulations but mean antenna efficiency is increased

\[ S(z) = \frac{I(0, z)}{2P/\pi \omega_d^2(z)} = \frac{\omega_d^2(z)}{\omega^2(z)} \left[ 1 + e^{-\frac{2}{\rho_T^2}} \right] \]

- Far Field– Beam is Near Gaussian in Shape at Moderate Truncation Ratio and Strehl is

\[ S(z) = \frac{I(0, z)}{2P/\pi \omega_d^2(z)} = \left[ 1 - e^{-1} \right]^2 \]
Far Field SAE Prediction

- Far Field Antenna Efficiency of SAE Model is Pretty Good Across All Truncation Values Requiring only a Small Correction Factor
Good Agreement With Far Field Misalignment Sensitivity

With $(\eta_T)^{0.25}$ factor in exponent argument
Comparison Of UGB, SAE, and CSAE Models vs Range

- SAE approximation is a significant improvement over the UGB model, but requires further correction to improve match to numerical results.
Corrected Strehl-based Antenna Efficiency (CSAE) Construction

- CSAE Analytic Model made of Three Components Properly Combined - see Summary Paper for Details

\[
\eta_{a,CSAE}(z) = \eta_T^2 \frac{(\lambda z)^2}{A_r} \frac{1}{\pi \omega_{eff}(z)},
\]

\[
\omega_{eff}^2 = \frac{\omega_{eff,S}^2 + \omega_{eff,l}^2}{2} \quad \text{for} \quad z_{Fe} \leq z_b
\]

\[
\omega_{eff}^2 = \frac{\omega_{eff,S}^2 + \omega_{eff,g}^2}{2} \quad \text{for} \quad z_{Fe} > z_b
\]

\[
\omega_{eff,S}(z) = \frac{\eta_T}{f_c S(z)} \omega_d^2(z)
\]

\[
\omega_{eff,l} = \frac{1 - \exp \left(-\frac{2}{\rho_T^2}\right)}{1 + \exp \left(-\frac{2}{\rho_T^2}\right)} \omega^2(z)
\]

\[
\omega_{eff,g} = \omega_0^2 \left[ \frac{1}{M^2} \left(1 - \frac{z}{f}\right)^2 + M^2 \left(\frac{Z}{z_R}\right)^2 \right]
\]
Impact of Aberrations

\[ \eta_{a,ATGB} \approx f_c \eta_T \rho_t^2 S = \frac{f_c \rho_t^2 \left(1 - e^{-\frac{2}{\rho_t^2}}\right)}{\left[1 + (2\pi \sqrt{12} \rho_t^2 \sigma_{df})^2\right]} \left[(1 + e^{-\frac{2}{\rho_t^2}}) - 2e^{-\frac{1}{\rho_t^2}} \cos[2\pi \sqrt{12} \sigma_{df}] \right] \]
Summary

• Developed Improved Analytic Model For Estimation of Gaussian Beam Lidar Including Truncation and Aberrations

• Model is Significant Improvement over Untruncated Gaussian Beam Model Used by Most Lidar Researchers and Engineers

• Continuing to Improve the Model and Plan a More Complete Publication at a Later Date
BACKUP SLIDES
\[\eta_{a,\text{CSAE}}(z) = \eta_T^2 \frac{(\lambda z)^2}{A_r} \frac{1}{\pi \omega_{\text{eff}}^2(z)},\]

where

\[\omega_{\text{eff}}^2 = \frac{\omega_{\text{eff},S}^2 + \omega_{\text{eff},l}^2}{2} \quad \text{for} \quad z_F \leq z_b\]

\[\omega_{\text{eff}}^2 = \frac{\omega_{\text{eff},S}^2 + \omega_{\text{eff},g}^2}{2} \quad \text{for} \quad z_F > z_b\]

\[\omega_{\text{eff},l} = \frac{1 - \exp \left( -\frac{2}{\rho_{\text{T}}^2} \right)}{1 + \exp \left( -\frac{2}{\rho_{\text{T}}^2} \right)} \omega^2(z)\]

\[\omega_{\text{eff},g} = \omega_0^2 \left[ \frac{1}{M^2} \left( 1 - \frac{z}{f} \right)^2 + M^2 \left( \frac{z}{z_R} \right)^2 \right]\]

\[\omega_{\text{eff},s}(z) = \frac{\eta_T}{f_c S(z)} \omega_{\text{d}}^2(z)\]

\[\omega^2(z) \approx \omega_0^2 \left( 1 - \frac{z}{F} \right)^2 + \left( \frac{\lambda z}{\pi \omega_o} \right)^2\]

\[\omega_{\text{d}}^2(z) = \lambda z / \pi \omega_o\]

\[M^2 = \sqrt{\left( 1 - \exp \left( -\frac{2}{\rho_{\text{T}}^2} \right) \right) / f_c \left( 1 - \exp \left( -\frac{1}{\rho_{\text{T}}^2} \right) \right)^2} \]